Introduction to Nonmonotonic Reasoning
Michal Peliš
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`http://web.ff.cuni.cz/~pelis`

1 Consequence relations

background-logic: classical propositional (predicate) logic ($\mathcal{L}$)

Consequence relation is a set of pairs $\langle\Gamma, \Delta\rangle$ where Γ and Δ are sets of formulas (Γ is a set of premises).

$\langle\Gamma, \{\varphi\}\rangle$ where φ is a formula

- *syntactical* consequence relation ($\Gamma \vdash \varphi$)

$$\psi_0, \psi_1, \ldots, \psi_n = \varphi$$

  - $\psi_i \in \Gamma$
  - $\psi_i \in \mathcal{L}$
  - ψ_i is a result of an application of a rule to some $\psi_k, \ldots, \psi_l$ (where $k, \ldots, l \leq i$)

- *semantical* consequence relation ($\Gamma \models \varphi$)

Completeness theorem: $\Gamma \vdash \varphi$ iff $\Gamma \models \varphi$

$$Cn\Gamma = \{\varphi : \Gamma \models \varphi\} \text{ OR } \{\varphi : \Gamma \vdash \varphi\}$$

2 Important properties of Cn

- reflexivity

$$\Gamma \subseteq Cn\Gamma$$

- cumulative transitivity (Cut)

$$\Gamma \subseteq \Delta \subseteq Cn\Gamma \quad \Rightarrow \quad Cn\Delta \subseteq Cn\Gamma$$

- monotony

$$\Gamma \subseteq \Delta \quad \Rightarrow \quad Cn\Gamma \subseteq Cn\Delta$$

3 Example of common reasoning

```
Smith entered the office of his boss.
He was nervous.
```

1. **typically**: (x enters the office of his boss) $\rightsquigarrow$ (x is nervous)

2. **infer**: *Smith is nervous.* (*)
3. **new info**: `After all, he did not want to lose his best employee.`
4. **then**: (*) is wrong

4 Where to use nonmonotonic reasoning

- legal reasoning
- diagnosis
- natural language understanding
- intelligent tutoring systems

5 Ways of getting more

1. Add new (believed) assumptions.
2. Add preferential relations among models.
3. Add new rules.

5.1 New assumptions

- Γ — background knowledge
- B — "what is believed"

$$\varphi \in Cn^B\Gamma \text{ iff } \Gamma \cup B \models \varphi$$

(monotonic)

5.2 Default assumptions

$\boxed{\text{Consistency: } M \text{ is CONS iff } CnM \neq Fla.}$

Let $B_1 \subseteq B$. B_1 is MaxCONS wrt Γ iff

1. B_1 is CONS wrt Γ,
2. $\forall D \subseteq B(B_1 \subset D \Rightarrow D$ is not CONS wrt $\Gamma)$.

$\boxed{C^B\Gamma = \cap\{Cn(\Gamma \cup B_1) : B \supseteq B_1 \in \text{MaxCONS wrt } \Gamma\}}$

- nonmonotony
- cautious monotony

$$\Gamma \subseteq \Delta \subseteq C^B\Gamma \Rightarrow C^B\Gamma \subseteq C^B\Delta$$

5.3 Preferred models (valuations)

- $v \models \varphi$ iff $v(\varphi) = 1$
- V — set of all valuations
- $\langle V, \prec \rangle$ — $\prec$ is a strict partial ordering on V
- $v \models_{\prec} \varphi$ iff
  1. $v \models \varphi$,
  2. $\forall v_1 (v \prec v_1 \Rightarrow v_1 \not\models \varphi)$.

$$\boxed{C^{\langle V, \prec \rangle}\Gamma = \{\varphi : \forall v (v \models_{\prec} \Gamma \Rightarrow v \models \varphi)\}}$$

- nonmonotony
- *not* cautious monotony
  - cautious monotony:

$$\Gamma \subseteq \Delta \subseteq C^B\Gamma \Rightarrow C^B\Gamma \subseteq C^B\Delta$$

- *not* consistency preservation
  - consistency preservation:

$$\Gamma \in \text{CONS} \Rightarrow C^B\Gamma \in \text{CONS}$$

5.4 Default rules

$$\frac{\text{goToWork} : \text{useBus}}{\text{useBus}}$$

$$\delta = \frac{\varphi : \psi_1, \ldots, \psi_n}{\chi}$$

δ is *applicable* to E ($E = CnE$) iff $\varphi \in E$ and $\neg\psi_i \notin E$ (for all $i \in \{1, \ldots, n\}$)
Default theory: $\langle \Gamma, D = \{\delta_0, \delta_1, \ldots\} \rangle$
Extension (operational definition)

5.4.1 Processes and extensions of a default theory $\langle \Gamma, D \rangle$

E	Π	test 1	Out	test 2
$Cn\Gamma$	δ_0	appl. to E?	$\neg\psi_i$	$E \cap Out = \emptyset$?
$Cn(\Gamma \cup \{\chi\})$	δ_1	$\ldots$	$\ldots$	$\ldots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

E is *extension* of $\langle \Gamma, D \rangle$ iff

1. all applicable δ's were applied (process Π is *closed*),
2. $E \cap Out = \emptyset$ (process Π is *successful*).

5.4.2 Examples

1. $\Gamma = \emptyset$, $D = \{\frac{:\alpha}{\neg\alpha}\}$
2. $\Gamma = \{\neg p, q\}$, $D = \{\frac{q:\neg r}{p}\}$
3. $\Gamma = \emptyset$, $D = \{\delta_0, \delta_1\}$
   - $\delta_0 = \frac{:p}{\neg q}$
   - $\delta_1 = \frac{:q}{r}$
4. $\Gamma = \{\text{green, drivLic}\}$, $D = \{\delta_0, \delta_1\}$
   - $\delta_0 = \frac{\text{green}:\neg\text{likeCar}}{\neg\text{likeCar}}$
   - $\delta_1 = \frac{\text{drivLic}:\text{likeCar}}{\text{likeCar}}$

5.4.3 Properties

- minimality of extensions

$$E, F \text{ extensions of } \langle \Gamma, D \rangle.\ E \subseteq F \Rightarrow E = F$$

- consistency preservation
- nonmonotony (in both Γ and D)

6 References

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3. Meyer, J.-J. Ch. and van der Hoek, W. *Epistemic Logic for AI and Computer Science.* Cambridge Univ. Press, 1995.
4. Brewka, G. *Nonmonotonic Reasoning.* Cambridge Univ. Press, 1991.