Epistemic Erotetic Search Scenarios

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Abstract

In this paper we provide an epistemic interpretation of erotetic search scenarios. We introduce the concepts of askability and of epistemic erotetic implication. We discuss epistemic erotetic search scenarios for an agent interacting with an information source. The proposed framework allows for modelling the process of problem solving with questions which is relative to agents' knowledge.

Keywords: Inferential Erotetic Logic (IEL), Dynamic Epistemic Logic (DEL), questions, e-scenarios, agenda

Introduction

As a point of departure in this paper we take the ideas from A. Wiśniewski's Inferential Erotetic Logic (IEL) [16, 20], however we relay on their epistemic interpretation proposed and discussed in details in [11, 10]. Such an approach allows us to discuss problem-solving and questioning agenda in the context of agents' interaction and takes into account agents' knowledge.

The pragmatic intuition underlying the concept of erotetic search scenarios (e-scenarios for short) is that an e-scenario

> (...) provides information about possible ways of solving the problem expressed by its principal question: it shows what additional data should be collected if needed and when they should be collected. What is important, an e-scenario provides the appropriate instruction for every possible and just-sufficient, i.e. direct answer to a query: there are no "dead ends". [19, p. 110].

E-scenarios has proven to be a powerful logical tool for modelling cognitive goal-directed processes (cf. [18, 17, 21], [13], [14]). Erotetic search scenarios were also employed in the field of problem solving (see [3] and Section 1 of this paper). What is more, the pragmatic account of the scenarios was used in the Turing test adequacy debate (see [4, 5] and [8]). See also [12] for the connection between e-scenarios and proof theory. E-scenarios were also used as a basis for the procedure of generating co-operative responses useful for interfaces of information systems—see [7] and [6]. The procedure presented in [7] was implemented in Prolog and is available for download.[1]

This paper is structured as follows. We discuss similarities and differences between our approach and already existing frameworks for agents using questioning agendas in

[1] The program *COOP Q-RESPONSES* is available at the page of INTQUESTPRO project: `https://intquestpro.wordpress.com/resources/software/`. At the same page the Prolog program *CPC e-scenarios*, that generates atomic e-scenarios for yes-no questions is also available.

Section 1. Afterwards, in Section 2, we analyse some motivational examples explaining our main intuitions and types of epistemic situations that we are addressing in our framework. Section 3 introduces elementary concepts of epistemic logic with questions, such as askability of a question, answerhood conditions and epistemic erotetic implication. Section 4 covers epistemic erotetic search scenarios. We introduce basic epistemic scenarios and present how they are enveloped into questioning agendas—epistemic erotetic search scenarios. We also discuss, how the knowledge structure of an agent influences generation of epistemic erotetic search scenarios. The last two sections covers public announcement interpretation of epistemic erotetic scenarios (Section 5), summary of the discussed ideas and possible directions of their further developments, especially, an extension of the proposed approach to multi-agent settings, where a group of agents cooperatively solve a complex problem.

1 Related work

The framework presented here stems directly from IEL. Our idea of an epistemic erotetic scenario (e-e-scenario for short) is adapted from the concept of erotetic search scenario presented for the first time by Wiśniewski in [18]. The key difference is that our scenarios are based on the notion of epistemic erotetic implication in contrast to erotetic implication of Wiśniewski. As we discussed in the introduction this allows us to consider the questioning agenda issue in the epistemic context. The core intuitions about the structure of search scenarios and their purpose remain the same.

A more straightforward inspiration from the IEL is [13] which presents how we may use e-scenarios for formal modelling of agents' hidden agendas for questioning. From this paper we take the idea that there are certain cases when an agent does not want (or simply cannot) reveal her initial question. Moreover, examples analysed in [13], show that e-scenarios used in such contexts offer a possibility of differentiation of questions posed by an interrogator to herself from the ones that actually should be asked to a person being interrogated. We address this issue in our framework. We also aim at clear differentiation of the factual data gathered by an agent and the inferences which she takes on this data. At this point, we should also mention [3], where a three-valued logic is employed in order to express the lack of information involved into problem-solving process. In the approach presented in this paper we exploit this idea. A situation when an agent encounters a lack of information after asking the initial question is a crucial point for our basic epistemic scenario—this is a point from which an agent starts developing her questioning agenda.

It is also worth mentioning another interesting approach to problem-solving *via* questioning, which stems from Hintikka's Interrogative Model of Inquiry framework. In [2] Genot introduces a framework for analysing questions in interaction based on Interrogative Games and the game theory. The main objective is to apply these in the field of Belief Revision Theory as presented in [9]. The idea is to investigate agent's questioning agenda and its modifications during information seeking process. Genot [2] presents this process in a form of a game between two agents, where two types of moves are allowed: deductive ones (when a questioning agent states something) and interrogative moves (when the agent asks whether something). We use a similar idea in our definition of epistemic erotetic search scenario. A full e-e-scenario is developed from the basic epistemic scenario (which is a form of the initial epistemic situation for our questioning agent) *via* expansion steps (deductive and erotetic ones). Moreover, the framework proposed in our paper allows for modelling agent's agendas for questioning and grasps the idea of dependence between knowledge of an agent and the agenda structure. What is more, due to well defined concepts of epistemic logic with questions, we can extend our framework into a multi-agent setting in order to model interaction of many agents collaborating on solving a given problem.

2 Motivation

Our main focus in this paper is to address a process of solving a problem with the use of questioning. It is often the case that we ask a question and we do not receive an answer to this question. Such a situation results in developing certain questioning agenda based on the initial question. In order to devise such an agenda, the initial question is replaced by some questions which are easier to answer. This may be observed in two examples retrieved from the British National Corpus [1].

EXAMPLE 1. A: *The Chairman of the County Council has just got food poisoning (pause) Where did he pick it up?*
A: *What restaurant was he in?*
A: *Have you ever inspected the restaurant?*
[KRP, 250–252][2]

EXAMPLE 2. A: *Question six (pause) okay for anybody who's interested in eating, as we are, pate de foie gras is made from what?*
A: *Right we'll be even more specific right, a help for ya, pate de foie gras is made from the liver of what?*
[KDC, 20–21]

In the example (1) the initial question is replaced with two more detailed ones, while in the example (2) the initial question seems to be too difficult to be answered right away, so the speaker A replaces it with a simpler question (asking about more specific information).

Of course we can imagine that the motivation of an agent which leads to modifying her initial question or even developing the whole questioning agenda may be more complex. Let us illustrate this by the following example.

EXAMPLE 3. *Carol just came to a party. She is obviously interested who else will be there. So she asks: "Are Andrew and Barbara also coming?". She does not get any answer. She thinks that people around her either do not now or took her too literally and if they do not know a full answer they rather keep silent than give at least some partial information. Or it might be too difficult to answer this question at this stage of the party. Carol does not give up and asks: "Is Andrew at the party?". "No" somebody replies. It is clear to Carol now, that if the answer were "Yes", it would make sense to ask about Barbara, but not now, because her question is already answered. She also notices that the one who replied "No" could have replied already the first question, if he bothered to think a bit.*

The situation might also be different. Coming to a party next week she knows very well, that Andrew will be at the party (because she had a secret date with him last night), but she would like to know if Barbara is there as well. However asking about Barbara directly might lead to some unwanted question like "Why is she interested only in Barbara? How does she know that Andrew is here?". Thus Carol decides to

[2]This notation indicates the BNC file (KRP) together with the sentence numbers (250–252).

solve the problem by asking the same question as the last week—this way hiding the information she has and getting the information she needs. Obviously, if it will again come to the second question, concerning Andrew (in the case nobody gives a direct answer) and this question would be answered positively, Carol gets a full answer and asking about Barbara does not give her any new information.

Carol's strategy in the example was rather straightforward:

1. Do not ask questions about things you know.
2. Ask directly of maximal information.
3. If you don't get any reply, ask some simpler question(s), which leads to the answer to your original question.
4. Try to infer the answer to the initial question on the basis of the obtained replies.

All three examples present an agent-questioner who wants to obtain an answer to her (initial/main) question. She asks that question and expects an answer from other agents. What follows after the question being asked influences our agent-questioner's epistemic state. The agent either gets a simple strightforward answer or no answer at all. These both situations will influence her next step in the questioning strategy.

In order to express intuitions underlying the interplay of questions and knowledge in a formal way we will now introduce epistemic logic with questions.

3 Epistemic logic with questions

Our basic framework will be the standard S5 epistemic logic supplemented with erotetic operators. The language consists of a countable set of atomic formulas $\mathcal{P} = \{p, q, ...\}$ and formulas defined by BNF as follows:

$$\psi ::= p \mid \neg\psi \mid \psi \to \psi \mid K\psi$$

We interpret the modality $K\psi$ as 'the agent knows that...'. We will use also the modality $\hat{K}$, i.e., 'the agent considers it possible that...', which is defined as a dual to K:

$$\hat{K}\psi =_{df} \neg K \neg\psi$$

In this paper we use the standard S5 semantics based on Kripke frames. A (Kripke) frame is an ordered pair $\langle W, R\rangle$ where W is a non-empty set of possible worlds and and the relation $R \subseteq W^2$ is an equivalence relation. A (Kripke) model $M = \langle W, R, V\rangle$ is a (Kripke) frame with a valuation function V, defined in the usual manner.

The satisfaction relation $\models$ is defined in the standard way:

- $(M, w) \models p$ iff $w \in V(p)$;
- $(M, w) \models \neg\psi$ iff $(M, w) \not\models \psi$;
- $(M, w) \models \psi_1 \to \psi_2$ iff $(M, w) \models \psi_1$ implies $(M, w) \models \psi_2$;
- $(M, w) \models K\psi$ iff $(M, v) \models \psi$, for each v such that wRv.

Other propositional connectives ($\wedge, \vee, \leftrightarrow$) are introduced in the standard way. The notions of satisfaction in a model or in a frame and validity are defined as usual.

3.1 Questions

Now, we introduce the language for S5Q as an extension of epistemic S5 by symbols of curly brackets $\{,\}$ and a question mark ?. A question is an expression of the following form:

$$?\{\alpha_1, \alpha_2, \dots \alpha_n\}$$

where $\alpha_1, \alpha_2, \dots, \alpha_n$ are formulas of S5Q, which we call *direct answers to the question*. An intended reading of a question of this form is: 'Is it the case that α_1 or is it the case that α_2 ... or is it the case that α_n?' The set of direct answers is finite, the formulas $\alpha_1, \dots, \alpha_n$ are syntactically distinct, and we require that there are at least two direct answers ($n \geq 2$).

In what follows, we use a metavariable Q (possibly with subscripts) for questions, and we write dQ for the corresponding set of direct answers. We will use the following abbreviations: $?\alpha$ denotes a simple yes-no question $?\{\alpha, \neg\alpha\}$, and $?|\alpha, \beta|$ denotes a binary conjunctive question of the form: $?\{\alpha \wedge \beta, \alpha \wedge \neg\beta, \neg\alpha \wedge \beta, \neg\alpha \wedge \neg\beta\}$.[3]

Askability Askability is the central notion for our framework. It specifies conditions under which an agent is justified to ask a question. The intuitions behind the askability are the following. If an agent (honestly) asks, for example, a question $?\{\alpha, \beta\}$ the addressee obtains the following information:

1. The agent does not know whether α or β.
2. The agent considers both α and β (epistemically) possible.
3. The agent expects that it is the case that α or it is the case that β (it is not epistemically possible that neither of them holds).[4]

We illustrate this idea with a simple example (see [10, p. 62]). There is a group of three friends: Anne, Bill, and Catherine. Each of them has one card and nobody can see the cards of the others. One of the cards is the Joker and everybody knows this fact. At this point Catherine. who does not have it, asks: *Who has the Joker: Anne, or Bill?*. This question has two-element set of direct answers, namely {Anne has the Joker, Bill has the Joker}. Asking her question, Catherine expresses that: (1.) she does not know the answer to her question; (2.) she considers the answers to be possible; and (3.) she presupposes that either Anne has the Joker or Bill has it.

These intuitions behind askability are formalized in the following definition:

DEFINITION 1 (Askability)**.** *We say that a question $Q = ?\{\alpha_1, \alpha_2, \dots \alpha_n\}$ is* ask-able *in a state (M, w) (in symbols $(M, w) \models Q$) iff*

1. $(M, w) \not\models K\alpha_i$, for each $\alpha_i \in dQ$ (non-triviality)

2. $(M, w) \models \hat{K}\alpha_i$, for each $\alpha_i \in dQ$ (admissibility)

3. $(M, w) \models K(\alpha_1 \vee \alpha_2 \vee \dots \vee \alpha_n)$ (context)

Semantically the askability conditions mean that the set of states accessible from w by relation R, contains for each $\alpha \in dQ$ at least one state in which α holds (2.), but, at

[3] For a generalised definition of conjunctive questions see [12, p. 76].

[4] We allow that both α and β hold.

the same time, contains at least one state in which α does not hold (1.). Moreover, at least one direct answer α should hold in each state (3.).

As a result, the askability of $Q = ?\{\alpha_1, \dots \alpha_n\}$ in (M, w) is equivalent to

$$(M, w) \models (\neg K\alpha_1 \wedge \dots \wedge \neg K\alpha_n) \wedge (\hat{K}\alpha_1 \wedge \dots \wedge \hat{K}\alpha_n) \wedge K(\alpha_1 \vee \dots \vee \alpha_n). \quad (1)$$

It can be simplified to a conjunction of the context condition plus "extended admissibility" expressing that each direct answer is considered to be possible by the agent as well as its negation:

$$(M, w) \models [(\hat{K}\alpha_1 \wedge \hat{K}\neg\alpha_1) \wedge \dots \wedge (\hat{K}\alpha_n \wedge \hat{K}\neg\alpha_n)] \wedge K(\alpha_1 \vee \dots \vee \alpha_n).$$

Observe that for safe questions (most notably for yes-no questions), i.e., when $(\alpha_1 \vee \dots \vee \alpha_n)$ is a tautology, askability coincides with extended admissibility. The formulas above give us a reduction of (an askability of) a question to a standard S5 formula.[5]

Non-askable questions Obviously there are questions which are not askable. The simplest example are questions with tautologies/contradictions among direct answers. The language of S5Q allows for questions about knowledge and questions about questions, but in a single agent set-up these kinds of questions are not askable either. The reason for this is evident, because our background system is epistemic S5, our agent is fully introspective and hence fully aware of her knowledge and ignorance. In particular, the question $?\{K\varphi, \neg K\varphi\}$ is not askable for any φ.

PROPOSITION 1. *$(M, w) \not\models ?\{K\varphi, \neg K\varphi\}$, for any (M, w).*

Proof. If an agent knows φ (locally) in a state, i.e., $(M, w) \models K\varphi$, then the knowledge of φ is spread by the S5 axiom $K\varphi \to KK\varphi$ (positive introspection) and $\neg K\varphi$ cannot be true in any v, for wRv. This is in contradiction with non-triviality as well as admissibility conditions.

Similarly for the case that an agent does not know φ. Then we apply negative introspection axiom ($\neg K\varphi \to K\neg K\varphi$). $\square$

We can also speak about introspective agents with respect to questions. If a question is askable by an agent, then she knows this fact. The formula $(Q \leftrightarrow KQ)$ is valid in S5Q.

PROPOSITION 2. *The formula $(Q \leftrightarrow KQ)$ is valid in* S5Q.

Proof. Let $dQ = \{\alpha_1, \dots, \alpha_n\}$. For any (M, w), $(M, w) \models Q$ is equivalent to (cf. (1))

$$(M, w) \models (\neg K\alpha_1 \wedge \dots \wedge \neg K\alpha_n) \wedge (\hat{K}\alpha_1 \wedge \dots \wedge \hat{K}\alpha_n) \wedge K(\alpha_1 \vee \dots \vee \alpha_n)$$

$\Updownarrow$ from the duality of $\hat{K}$ and K

$$(M, w) \models (\neg K\alpha_1 \wedge \dots \wedge \neg K\alpha_n) \wedge (\neg K\neg\alpha_1 \wedge \dots \wedge \neg K\neg\alpha_n) \wedge K(\alpha_1 \vee \dots \vee \alpha_n)$$

$\Updownarrow$ apply positive as well as negative introspection

$$(M, w) \models (K\neg K\alpha_1 \wedge \dots \wedge K\neg K\alpha_n) \wedge (K\neg K\neg\alpha_1 \wedge \dots \wedge K\neg K\neg\alpha_n) \wedge KK(\alpha_1 \vee \dots \vee \alpha_n)$$

[5] Originally, the definition of askability was designed for the broader set of formal systems. Especially, for systems in which there might be a difference between non-satisfaction ($\not\models$) of a formula and satisfaction of its negation. The general study of expressive power of epistemic languages can be found in [15, Chapter 8].

$\Updownarrow$ because of the monotonicity axiom $K(\varphi \wedge \psi) \leftrightarrow K\varphi \wedge K\psi$

$(M, w) \models K((\neg K\alpha_1 \wedge \cdots \wedge \neg K\alpha_n) \wedge (\neg K\neg\alpha_1 \wedge \cdots \wedge \neg K\neg\alpha_n) \wedge K(\alpha_1 \vee \cdots \vee \alpha_n))$

is equivalent to $(M, w) \models KQ$. □

COROLLARY 1. *The formula* $(Q \leftrightarrow \hat{K}Q)$ *is valid in* S5Q.

As a consequence of the previous results we obtain that a question about question is not askable for an agent.

PROPOSITION 3. $(M, w) \not\models ?(Q)$*, for any* (M, w).

Proof. If $(M, w) \models ?(Q)$, then $(M, w) \models ?\{Q, \neg Q\}$. From Definition 1, both $(M, w) \models \hat{K}Q$ and $(M, w) \models \hat{K}\neg Q$. But, from Corollary 1 and Proposition 2, $(M, w) \models \hat{K}Q$ excludes that also $(M, w) \models \hat{K}\neg Q$. □

Let us note that questions about questions and questions about knowledge of other agents are possible and useful in a multi-agent set-up, see, e.g., [10]. For the purposes of this paper (single-agent set-up) we shall be considering only declarative formulas without modal operators among the direct answers to questions.

Answerhood conditions We assume that after an agent asked a question, she expects an answer. Below we specify the *answerhood conditions*, i.e., when we say that a question is answered in a state (for an agent):

1. A question Q is *(completely) answered* in a state (M, w) iff there exists $\alpha \in dQ$ such that $(M, w) \models K\alpha$.
   A question is completely answered if the agent knows at least one of the direct answers.

2. A question Q is *partially answered* in a state (M, w) iff there exists $\alpha \in dQ$ such that $(M, w) \models K\neg\alpha$.
   A question is partially answered if the agent knows that one of the direct answers is not true.

Answerhood conditions are based on the violation of the first two conditions in the definition of askability (Definition 1). In particular this means that questions completely or partially answered in a state are not askable in this state.

3.2 Epistemic erotetic implication and erotetic entailment relation

In what follows we introduce the notion of *epistemic erotetic implication* (hereafter e-e-implication) which describes how one can move from one question to another preserving askability. This leads us to the concept of a search scenario employing e-e-implication.

The notion of *epistemic erotetic implication* is a counterpart to the *erotetic implication* (e-implication) from Inferential Erotetic Logic [16, 20]. In IEL, erotetic implication is a semantic relation between a question, Q, a (possibly empty) set of declarative well-formed formulas, Γ, and a question, Q_1 (where Q is called an *interrogative premise* or simply *initial question*, the elements of Γ are *declarative premises* and the question Q_1 is the *conclusion* or the *implied question*—see [20, p. 51–52]). The core intuition behind e-implication may be expressed as follows. Let us imagine an agent who is trying to solve a certain (possibly) complex problem. The problem is expressed by her initial question. We assume that the agent does not have resources to answer the initial question on her own. Thus the initial question has to be processed. This processing is aimed at replacing the initial question with the one which is—possibly—easier to answer and such that each answer to it is useful in answering the initial question. The auxiliary question obtained as a result of the processing should have certain characteristics. First of all, it should stay on the main topic. In other words, no random questions should appear here. However, the main characteristic that we are aiming at here is that the answer provided to the auxiliary question should be at least a partial answer to the initial question (i.e. it should narrow down the set of direct answers to the initial question, see [20, p. 43]). It should bring our agent closer to solving the initial problem.

IEL provides the following conditions of validity for such an erotetic inference (see [20, p. 52–53]) making the described intuitions more precise:

1. If the initial question has at least one true answer (with respect to the underlying semantics) and all the declarative premises are true, then the question which is the conclusion must have at least one true answer.

2. For each direct answer B to the question which is the conclusion there exists a non-empty proper subset Y of the set of direct answers to the initial question such that the following condition holds:
   (♣) if B is true and all the declarative premises are true, then at least one direct answer $A \in Y$ to the initial question must be true.

The epistemic erotetic implication addresses the same intuitions. However, it takes the askability as a central notion. What is ensured by e-e-implication is a 'transfer of askability'. As such it does allow to tackle the idea of justification for certain erotetic moves. Whether an agent can ask a question, depends on the agent's knowledge. Such a solution suits our needs for epistemic erotetic search scenarios. One of drawbacks of using askability for our framework is that it does not allow for tackling the goal-directness of the original notion of e-implication as it is expressed by the second condition above. This has certain consequences for e-e-scenarios construction, where such a feature has to be imposed on the level of rules of construction (while in the Wiśniewski's framework it results from underpinning search scenarios with erotetic implication).

We start with the notion of a pure epistemic erotetic implication where we do not assume anything about agent's background knowledge. In epistemic approach to questions, knowledge of an agent is given implicitly—by a particular state of an epistemic (Kripke) model. However, in the proposed framework of epistemic erotetic scenarios it is much more natural to work with agent's knowledge explicitly.

DEFINITION 2 (Pure e-e-implication)**.** *A question Q_1* (purely) e-e-implies *a question Q_2 in a state (M, w) iff whenever Q_1 is askable in (M, w), Q_2 is askable as well. Formally:*

$$(M, w) \models (Q_1 \rightarrow Q_2) \textit{ iff } (M, w) \models Q_1 \textit{ implies } (M, w) \models Q_2.$$

We say that Q_1 (purely) entails *Q_2 iff $(M, w) \models (Q_1 \rightarrow Q_2)$ for each state w and and each model M (the formula $(Q_1 \rightarrow Q_2)$ is valid).*

Let us stress that the implication used in Definition 2 is a standard implication of standard epistemic logic.[6]

Epistemic erotetic implication has an expected property—transitivity:

COROLLARY 2 (Transitivity of pure implication)**.** *If* $(M, w) \models Q_1 \rightarrow Q_2$ *and* $(M, w) \models Q_2 \rightarrow Q_3$*, then* $(M, w) \models Q_1 \rightarrow Q_3$*, for any* (M, w)*.*

Let us now introduce few examples of valid pure e-e-implications which we use later in this paper (more examples can be found in [10, chapter 3.3]).

- Questions $?\alpha$ and $?\neg\alpha$ have the same askability conditions which is reflected by the following valid formulas:

$$?\alpha \rightarrow ?\neg\alpha \text{ and } ?\neg\alpha \rightarrow ?\alpha \tag{2}$$

- A question $?\{\alpha_1, \ldots, \alpha_n\}$ e-e-implies each yes-no question based on its direct answers:

$$?\{\alpha_1, \ldots, \alpha_n\} \rightarrow ?\alpha_j \text{ , for each } j \in \{1, \ldots, n\} \tag{3}$$

- A conjunctive question e-e-implies simple yes-no questions based on one of its conjuncts:

$$?|\alpha_1, \ldots, \alpha_n| \rightarrow ?\alpha_j \text{ , for each } j \in \{1, \ldots, n\} \tag{4}$$

As we pointed out, in the context of epistemic erotetic search scenarios we need an epistemic erotetic implication with explicit representation of the part of agent's knowledge 'relevant' to a particular question. This leads us to the definition of a generalized e-e-implication, i.e., implication which holds with respect to an auxiliary set of formulas.

First, we define what it means that a question is askable (for an agent) with respect to some auxiliary set of formulas.

DEFINITION 3. *We say that a question* Q *is* askable in (M, w) wrt Γ *iff* $(M, w) \models \{K\gamma \mid \gamma \in \Gamma\}$ *and* $(M, w) \models Q$*.*

Let us stress that Γ need not contain all of the agent's knowledge and, generally, need not be deductively closed. The definition only requires, that it is consistent (explicitly) and that it does not provide neither complete nor partial answer to Q (implicitly). The intended reading of Γ is that it explicitly represents part of agent's knowledge (in a given state) which is related to the question Q.

DEFINITION 4 (Epistemic erotetic implication)**.** *A question* Q_2 *is e-e-implied by* Q_1 *with respect to* Γ *in a state* (M, w)*, we write* $(M, w) \models \left(Q_1 \underset{\Gamma}{\rightarrow} Q_2\right)$*, iff*

if Q_1 *is askable in* (M, w) *wrt* Γ*, then* Q_2 *is askable in* (M, w) *as well.*

We say that Q_1 *implies* Q_2 *with respect to* Γ *and write* $\left(Q_1 \underset{\Gamma}{\rightarrow} Q_2\right)$ *iff*

$(M, w) \models \left(Q_1 \underset{\Gamma}{\rightarrow} Q_2\right)$ *for every* w *and* M*.*

[6] For broader discussion on the issue of implication between erotetic formulae see [10] and [20].

Strictly speaking $\left(Q_1 \underset{\Gamma}{\to} Q_2\right)$ is not a formula of S5Q, it is a notational abbreviation in the same sense as $?Q$ is.

Let us, for example, consider Q_1 to be $?\alpha$, and Q_2 to be $?(\beta \wedge \gamma)$; then Q_1 implies Q_2 on the basis of $\Gamma = \{\alpha \leftrightarrow (\beta \wedge \gamma)\}$:

$$?\alpha \xrightarrow[\{\alpha \leftrightarrow (\beta \wedge \gamma)\}]{} ?(\beta \wedge \gamma).$$

The role of Γ is to represent (explicitly) not only knowledge, but also a lack of knowledge. The latter is important in the context of non-triviality condition for askability. For example, it is not valid in general that $?(\alpha \wedge \beta) \to ?\alpha$, the question $?(\alpha \wedge \beta)$ is askable in a state (M, w), only if $(\neg K \neg \alpha)$, the knowledge of $\neg \alpha$ makes the question non askable. We can write

$$?(\alpha \wedge \beta) \xrightarrow[\{\neg K \alpha\}]{} ?\alpha.$$

Generally, whenever an agent considers both α and $\neg \alpha$ possible, then

$$?(\alpha \circ \beta) \xrightarrow[\{\neg K \alpha, \neg K \neg \alpha\}]{} ?\alpha \tag{5}$$

where $\circ$ is any of the connectives: $\wedge$, $\vee$, $\to$, $\leftrightarrow$. We will make use of this fact later.

We can interpret our examples of e-e-implication (1–5) as patterns of how to move from 'more complex' questions to 'less complex' questions.[7] This seems to be very useful in many cases. In Section 2, we saw some informal applications of such a move. Another example is when we ask information sources with restricted languages (e.g., query languages for databases) where we need decompose complex questions in order to make them 'understandable' for the addressee. Sometimes we want to conceal the complex question because asking it publicly could reveal too much about our knowledge and ignorance, e.g., in the case of police investigations (see, e.g., [13]). This viewpoint brings us again to the idea of erotetic search scenarios in epistemic logic. A general idea is to control agent's effective communication starting with a general question $?\{\alpha_1, \ldots, \alpha_n\}$ which is fully decomposed in atomic yes-no questions if necessary.

Nonetheless, before we introduce the idea of epistemic erotetic search scenarios, let us list some important properties of e-e-implication. Especially now, it is useful to emphasise that sets of formulas used in e-e-implication is our way to display (explicitly) knowledge relevant to the askability of corresponding question.

Erotetic epistemic implication is 'locally' as well as 'globally' monotonic:

PROPOSITION 4 (Monotonicity)**.** *Let Γ, Δ be sets of declarative formulas.*

1. *If $(M, w) \models \left(Q_1 \underset{\Gamma}{\to} Q_2\right)$, then $(M, w) \models \left(Q_1 \xrightarrow[\Gamma \cup \Delta]{} Q_2\right)$.*
2. *If Q_1 implies Q_2 wrt Γ, then Q_1 implies Q_2 wrt $(\Gamma \cup \Delta)$.*

Proof. 1. Suppose $(M, w) \models \{K\varphi \mid \varphi \in (\Gamma \cup \Delta)\}$ and $(M, w) \models Q_1$. If, simultaneously, $(M, w) \models \left(Q_1 \underset{\Gamma}{\to} Q_2\right)$, we obtain $(M, w) \models Q_2$. Because $(M, w) \not\models Q_2$ means that $(M, w) \not\models \{K\varphi \mid \varphi \in \Gamma\}$ or $(M, w) \models Q_1$, but it is not possible.

[7] Compare the motivation for the term *reducibility* in [16] and (especially in this context) in [10].

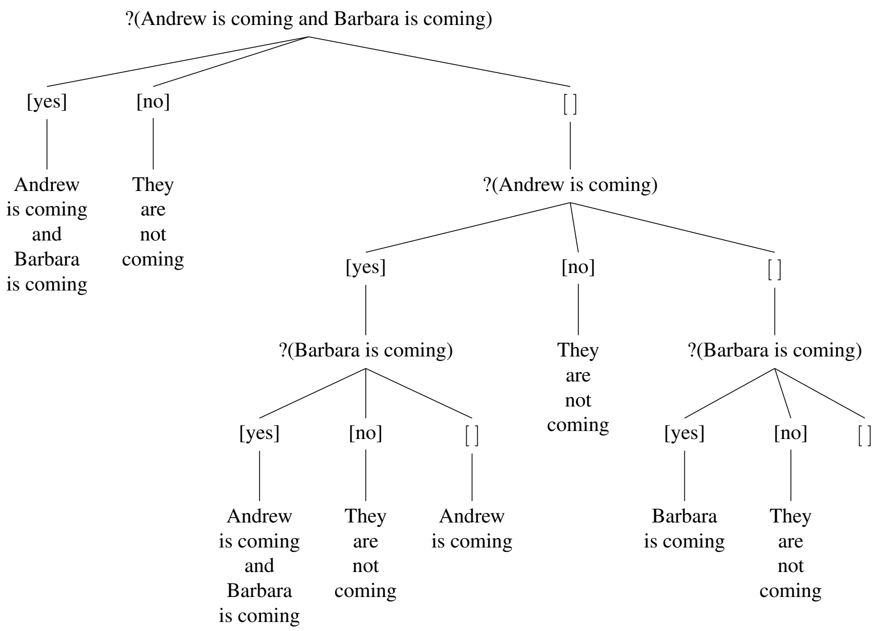


Figure 1: Representation of the possible courses of events for Example 3

2. Follows from 1.

□

Monotonicity can be used for the proof of a form of transitivity of e-e-implication with auxiliary sets of formulas:

PROPOSITION 5 (Transitivity)**.** *If* $(M, w) \models \left(Q_1 \underset{\Gamma}{\rightarrow} Q_2\right)$ *and* $(M, w) \models \left(Q_2 \underset{\Delta}{\rightarrow} Q_3\right)$, *then* $(M, w) \models \left(Q_1 \underset{\Gamma\cup\Delta}{\longrightarrow} Q_3\right)$.

Proof. If $(M, w) \models \{K\varphi \mid \varphi \in (\Gamma\cup\Delta)\}$ and $(M, w) \models Q_1$, then, from Proposition 4, $(M, w) \models \left(Q_1 \underset{\Gamma\cup\Delta}{\longrightarrow} Q_2\right)$ as well as $(M, w) \models \left(Q_2 \underset{\Gamma\cup\Delta}{\longrightarrow} Q_3\right)$. Thus, $(M, w) \models \left(Q_1 \underset{\Gamma\cup\Delta}{\longrightarrow} Q_3\right)$. □

4 Epistemic erotetic search scenarios

Let us return to Example 3 presented in Section 2. The tree in Figure 1 represents all situations which could have happened. It covers

- possible relevant questions Carol could have asked (indicated by the question mark symbol);
- possible replies she could have gotten (represented within square brackets) including also lack of answers (represented as [ ]).

The story described in the Example 3 corresponds to one of the branches in the tree in Figure 1, but things might have been different. Carol could have obtained an immediate positive answer—the process was quick and fully informative, see the leftmost branch in Figure 1. In the case of immediate negative answer (the second branch from left), she knows what she wanted to know, but does not have full information. She does not know who is missing—Andrew, Barbara or both of them. Nonetheless, the initial question is not askable any more. Our tree represents not only ways to obtain a particular direct answer to the initial question, but also information the agent gained in the process of asking and answering. This is remarkable in the case of failure—the agent might not get an answer, but still Carol knows more than before and she can continue in asking some other questions following the similar strategy.

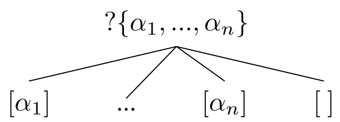
Assume, that an agent has some background knowledge Γ. Then there are two options: either the question is answered immediately (the agent obtains a direct answer) or she does not obtain any direct answer. Epistemic erotetic search scenario provides a pattern of questioning the agent should follow whenever she wants to solve a problem expressed by the question Q. A scenario simulates 'dialogue' between an agent and an external information source (other agent(s)). Informations coming from 'outside' are indicated by brackets [,].[8]

Formally, questions and premises involved in such a problem solving process are represented as a labelled tree or scenario for a question relative to the set of premises. In particular a question Q and an auxiliary set of epistemic $\mathsf{S5Q}$ formulas Γ, is represented as a labelled tree $\Sigma = (V, E)$ such that V is a finite set of nodes and $E \subseteq V^2$ is a set of edges.[9] The initial node r is labelled by Q and every other non-terminal node is labelled either by a formula in brackets or without brackets or by [ ]. Obviously scenarios correspond just to a certain kind of labelled trees and our task is now to specify them.

As we can see in the analysed example, the first step in the process of building a scenario is rather straightforward. The intuition behind this step is grasped by the notion of *basic epistemic erotetic search scenario* (basic scenario, for short).

DEFINITION 5 (Basic scenario)**.** *Let $Q = ?\{\alpha_1, ..., \alpha_n\}$ be a question. Basic epistemic erotetic search scenario for the question Q is a finite labelled tree Σ such that:*

1. *the root is labelled by Q;*
2. *there are n+1 successors of the root, n of which are labelled by $[\alpha_i], i = 1, ..., n$, respectively, and the n+1-st is labelled by [ ] indicating that the agent failed to get an answer.*



Let us remind here, that we are considering only questions with finite sets of direct answers, thus the resulting tree is finite.

When an agent is asking a question Q, and does not receive any immediate answer, she has to prepare a strategy which leads to an answer to the initial question or, at least, gains maximum of information the agent can obtain. To address this we introduce *expansions*, which is the way to develop a basic scenario into a (proper) e-e-scenario. An e-e-scenario is an expansion of the tree corresponding to a basic scenario using either a *deductive move* or an *erotetic move*.

[8]In Section 5 we introduce 'public announcement' interpretation for scenarios.

[9]Nodes will be denoted by r, x, y, z, etc.; r is for root-node.

Both moves crucially depend on knowledge the agent has collected up to the current state of the procedure. The following definition allows us to keep track of the information available to the agent in the current state of the procedure.

DEFINITION 6. *Let us take a tree $\Sigma = (V, E)$ for a question Q and an auxiliary set Γ labelled in the way described above. For $x \in V$ we define a set of auxiliary formulas as follows.*

1. $\Gamma(x) = \Gamma$ *if x is the root of Σ.*
2. *Otherwise there is a unique path (sequence of nodes) $\pi(x)$ in Σ from the root x_0 to the node x: $\pi(x) = \langle x_0, \dots, x_n, x \rangle$. Then $\Gamma(x) = \Gamma(x_n) \cup \{\varphi\}$ if x is labelled by $[\varphi]$, else $\Gamma(x) = \Gamma(x_n)$.*

$\Gamma(x)$ represents, in fact, agent's initial knowledge (Γ) and knowledge in the form of replies to auxiliary questions collected on the path from the root to x (indicated by nodes with square brackets). On the basis of this information the agent might be able to infer a reply to some question asked earlier.

DEFINITION 7 (Deductive expansion move)**.** *A terminal node x of a labelled tree Σ for Q and Γ is* deductively expandable *iff there is a question Q' on the path from the root to x such that $\Gamma(x) \models \alpha$ for some $\alpha \in dQ'$.*

The idea behind the *erotetic move* is to introduce an auxiliary question, which helps the questioner to find an answer to the initial question. Technically the agent checks at a node x whether there is a question that is e-e-implied (with respect to $\Gamma(x)$) by a certain question appearing earlier in the e-e-scenario. We should be careful, however. When we say that Q_1 implies Q_2 with respect to an auxiliary set $\Gamma(x)$, we can read $\Gamma(x)$ in two ways. In the strict reading $\Gamma(x)$ (together with askability conditions for Q_1) should *explicitly allow* asking the implied question, i.e. $\Gamma(x)$ and askability of Q_1 should entail the askability conditions of Q_2. This reading is captured by the notion of the (standard) e-e-implication. We can also admit a loose reading requiring just that $\Gamma(x)$ (together with askability of Q_1) *does not prevent us* from asking Q_2. This reading will be captured by the notion of a *default* e-e-implication. In a nutshell the strict reading means 'what is not explicitly allowed is forbidden', while the loose one means 'what is not explicitly forbidden is allowed'.

DEFINITION 8 (Default epistemic erotetic implication)**.** *A question Q_1 e-e-implies by default $Q_2 = ?\{\alpha_1, \dots, \alpha_n\}$ with respect to $\Gamma(x)$ in a state (M, w) iff*

1. *Q_1 is askable in (M, w) with respect to $\Delta = \Gamma(x) \cup \{(\neg K\alpha_1 \wedge \dots \wedge \neg K\alpha_n) \wedge (\hat{K}\alpha_1 \wedge \dots \wedge \hat{K}\alpha_n) \wedge K(\alpha_1 \vee \dots \vee \alpha_n)\}$, and*
2. $(M, w) \models \Big(Q_1 \underset{\Delta}{\rightarrow} Q_2\Big)$.

Then we write $(M, w) \models \left(Q_1 \xrightarrow[|\Gamma(x)|]{} Q_2\right)$.

In Definition 8 we require more than just e-e-implication. We say that the implying question is askable in a state (M, w) and this epistemic state is compatible with agent's knowledge collected in $\Gamma(x)$ together with askability conditions of Q_2, see Definition 3.

Using our notion of default epistemic erotetic implication we can define erotetic expansion as a move introducing a new question Q^* satisfying three conditions:

1. Q^* is implied by default by some previous question,
2. Q^* is not a repetition, and
3. Q^* is 'relevant', i.e., it helps to solve the initial question.

These conditions are formally captured by the following definition.

DEFINITION 9 (Erotetic expansion move)**.** *A terminal node x of a labelled tree Σ for Q and Γ is* erotetically expandable *by a question Q^* iff there is a question Q' which is a label of a vertex on the path $\pi(x)$ from the root of Σ to x such that*

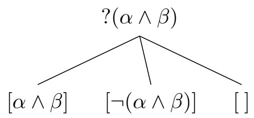
1. $\left(Q' \xrightarrow[|\Gamma(x)|]{} Q^*\right)$

2. $dQ^* \neq dQ''$*, for each Q'' on the path $\pi(x)$.*

3. there is at least one $\alpha \in dQ^*$ such that α together with $\Gamma(x)$ entails at least one answer (direct or partial) to some preceding question Q''. This means $\Gamma(x) \cup \{\alpha\} \models \beta$ or $\Gamma(x) \cup \{\alpha\} \models \neg\beta$ for some $\beta \in dQ''$.

The third condition of erotetic expansion move enables to prevent from useless inferences that are supported by 'actual epistemic state'. For example, if $(M, w) \models \hat{K}\alpha$ as well as $(M, w) \models \hat{K}\neg\alpha$, the question $?\alpha$ is also askable. But this question need not be useful for the solution of any previous question.

Before we define e-e-scenario, we show what is considered as an effective strategy. We use the tree presented in Figure 1 which is an (informal) e-e-scenario for the yes-no question *?(Andrew is coming and Barbara is coming)*. The process of reconstructing it in our formal language starts from the basic epistemic scenario. In the schema, α stands for *Andrew is coming* while β for *Barbara is coming*. There are no other premisses.



Now the expansion moves are employed. Let us start at the leftmost node labelled by $[\alpha \wedge \beta]$.

$$\Gamma([\alpha \wedge \beta]) = \{(\alpha \wedge \beta)\}$$

It is deductively expandable.

$$\Gamma([\alpha \wedge \beta]) \models (\alpha \wedge \beta),$$

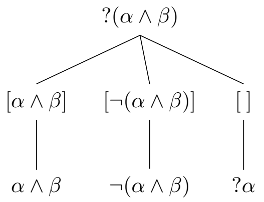
so the answer $\alpha \wedge \beta$ is placed in a child node.

Similarly for the $[\neg(\alpha \wedge \beta)]$-node. It is also deductively expandable, so the answer $\neg(\alpha \wedge \beta)$ is placed in a child node.

The rightmost node $[\,]$ is not deductively expandable. We check the possibility of applying an erotetic move. Since $\Gamma([\,]) = \emptyset$, we need a question which is (purely) e-e-implied or (at least) e-e-implied by default with respect to $\Gamma([\,])$ in the actual epistemic state. The only 'relevant' questions are $?\alpha$ or $?\beta$ (cf. the third condition in Definition 9). These are also e-e-implied by default, see (5):

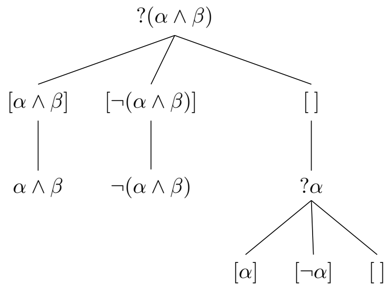
$$\left(?(\alpha \wedge \beta) \xrightarrow[|\Gamma([\,])|]{} ?\alpha\right) \text{ as well as } \left(?(\alpha \wedge \beta) \xrightarrow[|\Gamma([\,])|]{} ?\beta\right).$$

If we decide for $?\alpha$, the resulting structure is the following:

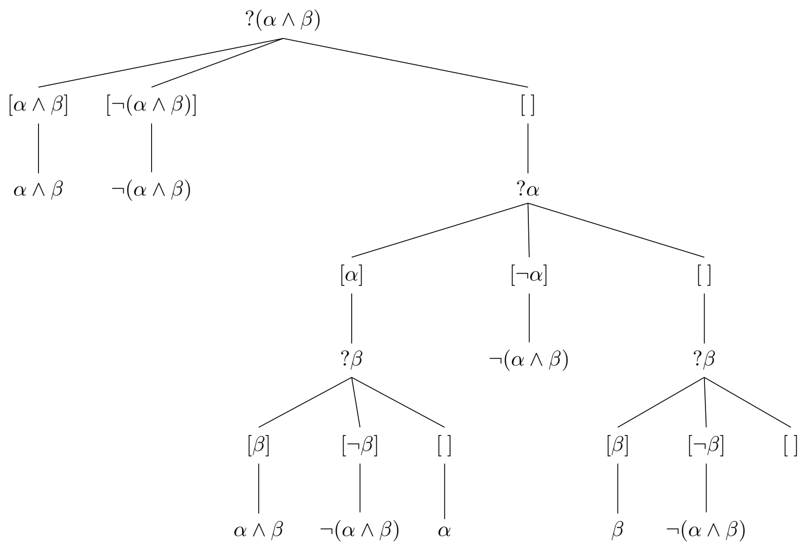


Now, it is not effective to expand nodes $(\alpha \wedge \beta)$ and $\neg(\alpha \wedge \beta)$, they contain direct answers to the initial question. We consider them as to be terminals.

For the node $?\alpha$ we can employ the basic epistemic scenario:



The newly introduced basic epistemic scenario for the question $?\alpha$ can be extended in the way analogous to the steps described above. The $[\alpha]$-node can be expanded deductively, we infer α from $\Gamma([\alpha])$. Then (or directly and effectively) we can use erotetic expansion by $?\beta$. The $[\neg\alpha]$-node can be expanded deductively and we obtain a direct answer to the initial question $\Gamma([\neg\alpha]) \models \neg(\alpha \wedge \beta)$. It terminates this branch. The [ ]-node can be extended erotetically by $?\beta$. The $?\beta$-nodes are managed similarly. The complete structure is presented below.



The leaves of the tree contain direct answers to the initial question or direct answers to auxiliary questions or [ ] which means that we have no information to answer any question in the tree.

A structure of the type presented above (i.e., with the empty set of initial premises Γ) is called *pure* e-e-scenarios, see, e.g, [20, p. 128].

Because we work in epistemic S5, empty set of initial premisses does not mean that the agent does not know anything. She knows at least all logical truths. As we mentioned above the role of $\Gamma(x)$ and $|\Gamma(x)|$ is rather to represent explicitly a part of agent's knowledge/ignorance related to a particular (initial or auxiliary) question.

The labelled tree presented in this section is built for the initial question of the form $?(\alpha \wedge \beta)$. We can introduce similar structures for other simple yes-no questions based on formulas with other connectives as the main ones. By the analogy to the erotetic search scenarios in IEL we will refer to them as *standard e-e-scenarios*. They are presented in the Appendix of this paper.

It is rather clear that questions askable with respect to a non-empty set of initial premises lead to more interesting e-e-scenarios.

Let us suppose that our agent wants to know whether it is the case that a piece of software is an open-source one. We know that it would be so, if and only if the source code of this software would be publicly available and permission to modify the code would be granted. This situation may be modelled within our approach.

We will represent the main question as $?o$. This question becomes the initial question of the agent's e-e-scenario. Moreover she knows that:

> The piece of software is an open-source one (o) if and only if its source code is publicly available (p) and permission to modify the code is granted (g).

We represent this piece of knowledge as $\Gamma = \{o \leftrightarrow (p \wedge g)\}$. Now we can present a model of a questioning agenda for our agent as the e-e-scenario presented in Figure 2.

Again, scenario consists of paths, which represent the possible ways in which the questioning might unfold (depending on the answers obtained). The leftmost path and the path next to it represent the situation when after our agent asks a question, a direct answer is provided immediately (which is represented by $[o], [\neg o]$ respectively). The rightmost path is more interesting. It is the case where after the initial question is asked, no answer is provided (which is represented by $[\ ]$). Then our agent decomposes the initial question with respect to her knowledge and gets the query of the form $?(p \wedge g)$; if no one knows the answer to $?o$, maybe it is the case that the new question might be resolved. One may observe that the answer to the new auxiliary question will provide our agent with the answer to the initial question (see the leaves of the e-e-scenario that are produced by deductive expansion moves).[10]

Epistemic erotetic search scenario will be defined as a labelled tree structure that is composed from some prototypical parts. These parts are connected via expansion moves taking into consideration effectiveness of a particular move. The starting point is clear, it is basic scenario. Nodes with direct answers in square brackets are deductively expandable. This move is considered as effective whenever the appropriate direct answers are inferred immediately. Another expansions are more free. Nonetheless, we can also follow some patterns as it is seen in our previous examples.

Deductive expansion moves (Definition 7) are preferred, especially if they provide a direct answer to an initial question. They bear answers to so far appeared questions and the main aim is to progress at nodes with answers to the initial question. We have to avoid of repetitions with the same results.

[10] In this example (Figure 2), we did not draw all lines (edges) to save space.

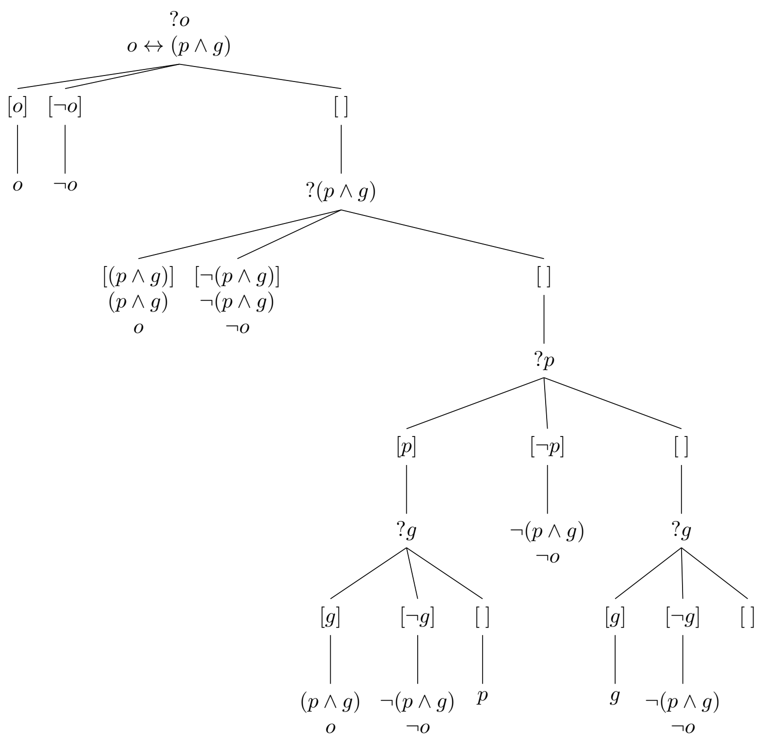


Figure 2: An example of e-e-scenario with a non-empty set of premises

In the case of erotetic expansion moves (Definition 9), we require that they are 'relevant' to the solution of an initial question. Fortunately, we do not have too many options, which questions can be considered as relevant. In Section 3.2, we introduced some prototypical e-e-implications, see (2), (3), (4), and (5). Especially, (3) says that any question Q entails yes-no questions based on direct answers to Q. The work with yes-no questions is crucial for our approach. In all examples we used variants of (5) where we moved from a compound yes-no question to more simple yes-no questions. These variants of (5), listed in Appendix, will be important components of scenarios.

DEFINITION 10 (epistemic erotetic search scenario)**.** *A labelled tree structure Σ is an* epistemic erotetic search scenario *for a question Q with respect to a set of formulas Γ iff*

- *The root is labelled by Q.*
- *If a node x is labelled by a question then there are n+1 successors of the root, n of which are labelled by $[\alpha_i], i = 1, ..., n$, respectively, and the $n+1$st is labelled by [ ] (cf. item 2 in Definition 5 – basic scenario).*
- *If a node x is labelled by a direct answer to Q without square brackets, then it has no child node.*
- *If a node x is not labelled by any question and there is x's child node y, then y is labelled in compliance with either*
  - *deductive expansion move (Definition 7), or*
  - *erotetic expansion move (Definition 9).*

Definition 10 defines which labelled tree structures are e-e-scenarios. Nonetheless, some of them need not be in a correspondence with our requirement of effectiveness. For example, it is useful to require that deductive expansions are preferred over erotetic ones and that expansions do not repeatedly produce the same formulas. In the following pseudo-code we define a procedure that describes the effective formation of an e-e-scenario for Q and Γ.

ProcEESS

- Input: Q, Γ
- Form basic scenario for Q (and Γ) according to Definition 5.
- Until all leafs (nodes without successors) are labelled by CLOSED, do
  - Start at the leftmost leaf x without CLOSED-label.
  - **ProcExpand** Form $\Gamma(x)$.
    * If x is deductively expandable by $\alpha \in dQ'$ such that Q' is the label of some $y \in \pi(x)$ (Definition 7), then form a new node $x + 1$ as a direct successor of x and label it by $\alpha \in dQ'$ for some $Q' \in \Gamma(x)$.
      ProcTest.
      Move to **ProcStep**.

    - ∗ If x is eroteticaly expandable by Q^* according to Definition 9, then form a new node $x + 1$ as a direct successor of x and label it by Q^*. Prefer pure e-e-implication over e-e-implication and prefer e-e-implication over default e-e-implication. If default e-e-implication is applied for compound yes-no questions, use patterns from Appendix.
      ProcTest.
      Move to **ProcStep**.
  - **ProcStep** If there is a leaf right from the actual one without CLOSED-label, move to it and do **ProcExpand**.

ProcTest If $(x + 1)$-label is the same as y-label, for $y \in \pi(x)$, then label $(x + 1)$ by CLOSED.

The basic step is always the formation of a basic scenario for an initial question (and a set of premises). **ProcEESS** ends whenever all leaves are labelled as CLOSED. This label is placed on nodes which are not labelled with a new label if we consider all previous labels of the path (**ProcTest**). In such a case, deductive or erotetic expansion move produces formulas that already occur on the path. We always start at the leftmost non-CLOSED node and after one-step expansion the procedure moves to the next non-CLOSED node in the right (**ProcStep**). In every non-CLOSED node the deductive expansion move is always preferred. If erotetic expansion move is applied, then valid e-e-implications are preferred, else the scenarios for $?(\alpha \wedge \beta)$, $?(\alpha \vee \beta)$, $?(\alpha \to \beta)$, and $?(\alpha \leftrightarrow \beta)$ are used from Appendix (**ProcExpand**).

5 Public announcement interpretation of e-e-scenarios

Our introductory examples present an agent asking questions and working with information obtained from outside. In fact, we drew a picture where the agent exchanges information with an external source. In logic, we can model such a process using the framework of *public announcement logic* (PAL), cf. [15] as an extension of our single-agent erotetic epistemic logic. The basic intuition behind PAL is that if a true statement φ is publicly announced by an agent in a group of agents, it becomes (except of some special cases) common knowledge in this group, i.e. everybody knows φ, everybody knows that everybody knows φ etc.

E-e-scenarios provide effective questioning strategies (agendas) that can be followed if someone wants to obtain an answer to an initial question. In reality, an agent follows some branch of the corresponding labelled tree that mirrors the actual exchange of information with the surrounding environment. There are two types of nodes in scenarios: with and without square brackets. The nodes with square brackets are understood as information coming from outside; these nodes indicate updates of agent's epistemic state. The other nodes represent agent's reasoning; these nodes do not bear 'new information' with respect to the actual epistemic state, they are not 'proper updates' for an agent. Nonetheless, every node can be understood as an action.

Nodes without square brackets are announcements of the agent-questioner directed to the environment (unspecified other agent(s)) and nodes with square brackets are announcements coming from outside (from the environment) to the agent. Thus, each path of a scenario can be understood as an update which is formed by a series of public announcements.

A node x labelled by either φ or $[\varphi]$, the formula φ (in the language of $\mathsf{S5Q}$) is considered as publicly announced in an actual epistemic state (M, w). Public announcement action changes (updates) this actual state to a 'new' state (M', w'). If $(M, w) \models \varphi$, then $w' = w$ and $M' = (W', R', V')$, where

- $W' = \{v \in W : (M, v) \models \varphi\}$,
- $R' = R \cap W'^2$,
- $V'(p) = V(p) \cap W'$.

There are no $\neg\varphi$-states in the updated model M'. The information φ becomes agent's knowledge in (M', w').

A node x labelled by [ ] ('no answer/information is available') does not bear any 'new fact', which would change agent's epistemic state. This has no direct counterpart in PAL, we can define in this case the new model M' to be the same as M.

We use single-agent PAL as a dynamic interpretation of individual knowledge update that is directed by e-e-scenarios.[11] Scenarios provide a unified questioning agenda (strategy) for an agent-questioner. The mentioned 'environment' is not considered as a genuine agent; it is just a (public) source of information.

This picture is important for our multi-agent approach, which is not presented in this paper. Nonetheless, let us mention our ideas informally. Scenarios are considered as individual questioning agendas (strategies). Agents in a group can share knowledge as well as e-e-scenarios, so they can deliver to other members appropriate information (individual knowledge) to solve their individual questions as well as (common) group questions. Scenarios can help to a cooperation in a group communication. The general idea was already presented in [10, 11], but there we did not apply any questioning agenda. In particular in [10, pp. 93–94], we proved formally:

> If an answer to Q is a distributive knowledge (in a group), then a finite sequence of statements corresponding to agents' individual knowledge (publicly announced) leads to the answer, which is a common knowledge in the group.[12]

This proposition postulates a finite sequence of public announcements, however, it does not give any method how to obtain it. Now, the method is given by e-e-scenarios—the required finite sequences are given by branches of scenarios.

We can illustrate the presented idea with a simple example. Let us imagine the following S5-epistemic model (reflexive loops are omitted) with the actual state s in the middle ($s \models \varphi, \neg\psi$):

$$\boxed{\begin{array}{ccccc} & a & & b & \\ \bullet & \longleftrightarrow & \bullet & \longleftrightarrow & \bullet \\ \varphi & & \varphi & & \neg\varphi \\ \psi & & \neg\psi & & \neg\psi \end{array}}$$

There are two agents (a and b). The first one, a, cannot distinguish between first two states, thus the question $?_a(\varphi \to \psi)$ is askable by a there. The second one, b, cannot distinguish between the second state and the third one and, there, the question $?_b(\varphi \to \psi)$ is askable by b. Hence the question:

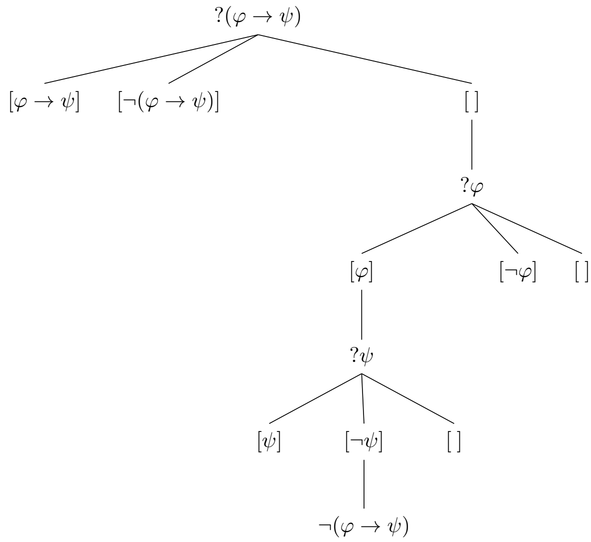
[11] Single-agent PAL is an unproblematic model of such a communication. There are no difficulties with *unsuccessful formulas* and *unsuccessful updates*. In multi-agent setting, knowledge about other agent's knowledge and group modalities (common and distributed knowledge) play important roles.

[12] For a definition of distributed and common knowledge see [15], an informal example is given below.

Is it the case that $(\varphi \to \psi)$?

is askable for both agents in s. None of them is able to answer it on her own, however, the answer to the question is distributive knowledge for them.[13] In the actual state s the agent a knows φ and b knows $\neg\psi$ there. The only thing they have to do, is to communicate effectively and publicly about their knowledge. The initial question can be announced by any of the agents. Since nobody can answer, no answer will be available. However, both of them can provide a 'useful' information according to the questioning agenda. The agent a provides φ and b provides $\neg\psi$.

For the question $?(\varphi \to \psi)$, they can use the e-e-scenario for implication. Their actual communication corresponds to one branch of the full scenario (see Appendix):



The full sequence of updates (node by node) is $?(\varphi \to \psi); ?\varphi; \varphi; ?\psi; \neg\psi; \neg(\varphi \to \psi)$.

6 Summary and further research

In this paper we proposed an epistemic interpretation of erotetic search scenarios with epistemic logic of questions as the underlying framework. What is novel in our approach is that we introduce the concept of the basic epistemic scenario, which addresses the first step of a questioning process. An agent asks a question and then expects an answer. Only in the case of lack of such an answer she will develop a questioning agenda. The intuition behind questioning agenda is grasped by epistemic erotetic search scenarios. We present how basic scenario is developed into a full fledged agenda by the expansion moves. We are labelling the external information by brackets notation. This allows us to show how this information (response to a question) is incorporated into an agent's agenda (transformed into answers or other questions).

Epistemic erotetic search scenario offers a full and effective strategy that is 'at hand' to every agent. The task for this strategy is to control asking and answering processes. In a multi-agent setting, strategies as well as initial questions and auxiliary

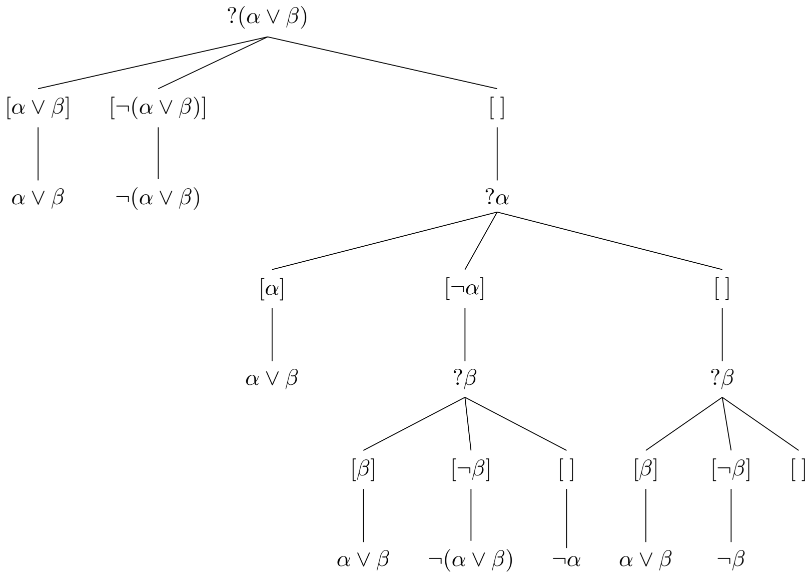
[13] χ is distributed knowledge for the group a, b, $(M, w) \models D_{\{a,b\}}\chi$ iff $(M, v) \models \chi$ for each v such that $w\ (R_a \cap R_b)\ v$, where R_a, R_b are accessibility relations of the agents a, b respectively.

sets of formulas can be shared among agents in a group. Scenarios can serve as a common strategy for a cooperative communication in a group.

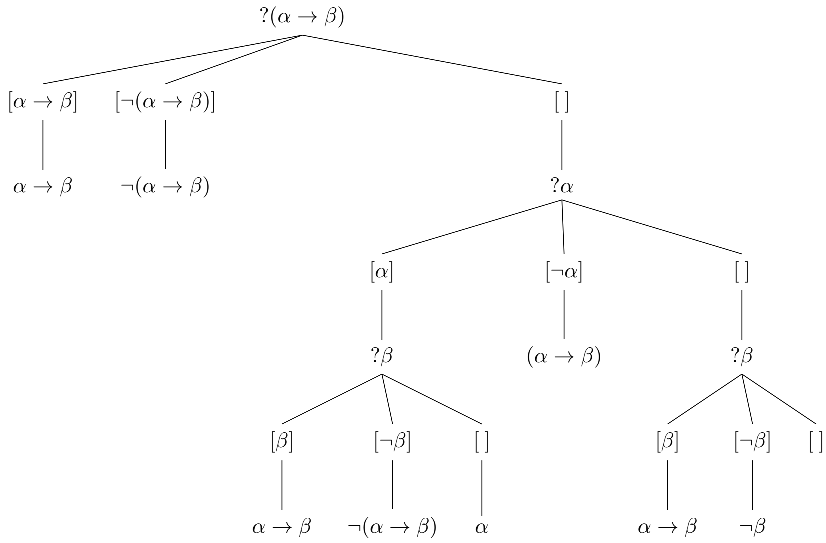
Future works will be focused on adding group dynamics to the presented picture. Especially we will extend the presented framework of e-e-scenarios for multi-agent settings (as it would be natural for using tool and concepts provided by epistemic logic of questions). This would allow us to grasp such interesting factors as knowledge exchange by public announcements.

Appendix

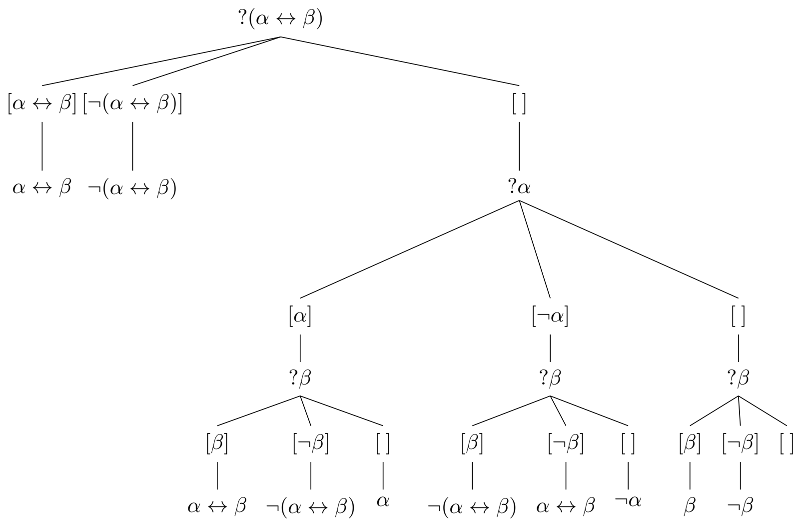
Standard e-e-scenarios for the connectives.
Disjunction



Implication



Equivalence



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